Handwritten Arithmetic Treatises in German (1400-1550). A First Assessment of the Sources Based on the Exemplary Corpus Held by the Austrian National Library

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The ERC Starting Grant project ARITHMETIC focuses on editing, describing, and analyzing handwritten arithmetic treatises in German from the 15th and 16th centuries. This paper is the result of a first preliminary study on the material that was conducted at the Austrian National Library, where an exemplary corpus of seven manuscripts was analyzed in detail. The results of this short study on a selective but representative corpus will serve as a basis for the upcoming work on the almost 140 manuscripts that form the complete research corpus of the ERC project. This article focuses on two main aspects in detail: On the one hand, the methodological approaches concerning the description of the manuscripts and the transcription of the arithmetic texts will be reviewed. In addition, the transcription software Transkribus, which serves as a support tool for the project, is used to generate a suitable model for handwritten text recognition (HTR) that will be used on large parts of the corpus. On the other hand, first hypotheses on the material will be tested and result in short analyses of the sources concerning questions on the process of translation from Latin to German, the relevance of the context handed down with arithmetic texts in miscellanies, the dichotomy of theoretical texts and practical teachings, and the importance of mnemotechnical tools in pragmatic texts of the Late Middle Ages.

Keywords: Late medieval mathematics, practical knowledge, translation culture (Latin–German), emergence of a German mathematical jargon, development of HTR models for arithmetic texts using Transkribus, description of arithmetic texts, late medieval mnemonics, late medieval knowledge culture

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Introduction

The ERC Starting Grant project ARITHMETIC[1] was kicked off in September 2022 at the Institute for Medieval Studies at the Austrian Academy of Sciences. The overall goal of this research project is to answer questions on the development of non-academic (mathematical) education in the Late Middle Ages, on the generation of a bourgeois knowledge society, and on the development of a vernacular mathematical jargon. Even though Latin was the lingua franca of medieval European scholarship, the rising independence of craftsmen, merchants and the bourgeoisie in the 15th and 16th centuries led to a need for education in the vernacular and therefore to the production and distribution of vernacular textbooks that are still largely unknown.[2] ARITHMETIC aims to focus on these early vernacular sources and will, in its first research step, collect, describe, transcribe, and digitally edit handwritten arithmetic texts in German with the goal of generating a detailed text base for further study. To get acquainted with the source material in question, a small exemplary corpus held by the Austrian National Library was examined in more detail. The goal was to determine whether these 10 manuscripts of arithmetic texts in German could inform us on what to expect when dealing with the complete corpus of roughly 140 manuscripts distributed over 60 cities throughout the world.

To sufficiently describe the overall rise of mathematical knowledge, the appropriation of the cultural technique of calculating with pen and paper and the enormous influence it had on society, culture, and education, it is necessary to look beyond the known Latin sources that are rooted in the academic circles. The fact that the rising independence of craftsmen, merchants and the bourgeoisie in the 15th and 16th centuries led to a different self-awareness and a need for education in the vernacular is still very much overlooked. This has led to a skewed, or at least incomplete picture of a late medieval knowledge society[3] in Europe, especially when we are looking at the proliferation of mathematical knowledge. We can tackle this problem by directing our attention to the vernacular sources, and by doing so, we will be able to contribute to and expand our current understanding of late medieval and early modern knowledge in a vernacular environment. This ERC project intends to focus on the German-speaking world as a first in-depth model study and will furthermore concentrate on the manuscript culture. We will pay attention to the time before the printing press helped

1 Further information on the project: www.oeaw.ac.at/imafo/forschung/historische-identitaetsforschung/projekte/arithmetic

2 Wedell, Numbers, 1205-1260, 1242-1243.

3 The late medieval knowledge society suffers from a poor reputation for scientific progress, intellectual achievement and innovation, even though it laid many of the foundations for the modern knowledge society. Moreover, the pursuit of a »circulation of knowledge« through diverse cultural and social levels is gradually replacing the earlier view of a top-down transmission of knowledge from a high level of production to a passive secondary audience. See, for example, Daum, Varieties of popular science; Lässig, History of knowledge; Roberts, Circulation of knowledge.

a wider audience to easily access all types of books, a time where we can already see a steep rise in vernacular scholarly texts written by hand all over Europe, clearly catering to an audience not rooted in the universities or monastic circles. For the arithmetic treatises alone, which are only a fraction of the didactic texts on mathematics in the 15th and 16th centuries, we can currently identify almost 140 manuscripts that contain texts in German and were loosely written between 1400 and 1550.[4]

This number is the result of in-depth research using the latest, often reworked and newly edited catalogs and databases.[5] In total we have collected data on over 300 manuscripts that contain handwritten mathematical texts in German that can be categorized as arithmetic, geometric or algebraic; computus treatises or astronomy were not even taken into consideration. Almost half of these manuscripts include content that can be classified as arithmetic and therefore deal with the art of calculating, to distinguish it from geometry, which is the art of measuring, and algebra, which is the art of calculating with closed sets. Arithmetic can be considered the foundation of mathematics, especially when it comes to the cultural technique of calculating with pen and paper built on the use of the Hindu-Arabic numerals and the positional notation system.[6] Without understanding the rules of arithmetic, no other number-based calculation can be conducted. This is one of the main reasons why arithmetical treatises are our first target of research; the basics of calculating need to be understood to use that knowledge continuously in adjacent fields.

The manuscripts from our corpus contain texts that deal with the art of calculating on different levels and through different types of texts: At first glance we need to distinguish between longer treatises that aim at giving thorough information on the art of arithmetic, like a reckoning book, and shorter texts that simply gather dense information or specifically teach very practical knowledge, like a reckoning example. This leads to a second distinction that

4 The initial cut-off date that was also given in the project-proposal was 1522, which is the year Adam Ries published his widely influential and often reprinted reckoning book (*Rechenung auff der linihen und federn in zal, maß und gewicht auff allerley handierung gemacht und zusamen gelesen durch Adam Riesen von Staffelstein Rechenmeyster zu Erffurdt im 1522 Jar).* The argument for that cut-off date was that after that print publication, the importance of printed reckoning books started to outweigh the handwritten tradition. That is true insofar as we can see a surge of printed textbooks; but what we overlooked at that time is that not only the printed but also the handwritten tradition is picking up: handwritten reckoning books are popping up everywhere and are an important factor in the distribution of mathematical knowledge in the 16th century. Therefore, we extended the cut-off date to c. 1550, a time when the printed reckoning books started to multiply and flood the German markets.

5 We started our research with an initial corpus of 80 manuscripts based on the Jordanus database hosted by the Bavarian Academy of Sciences (ptolemaeus.badw.de/jordanus/start). This database is the result of extensive research by Prof. Menso Folkerts and Gerhard Brey, who roughly 40 years ago collected data on mathematical, astronomical, and computational manuscripts of the Middle Ages in all Western languages with a strong focus on Latin manuscripts. The Jordanus database is incorporated into the Ptolemaios Latinus project (ptolemaeus.badw.de/start) currently undertaken by the Bavarian Academy of Sciences, but it has not been updated since the 1990s. This means that the Jordanus database, due to its outdated information, can only be considered a starting point for future research. Relying on new catalogs and databases, we were able to add over 60 manuscripts to our corpus and almost doubled the initial research corpus.

6 For more information on the history of mathematics with a focus on the development of mathematical theory and practice in medieval Europe, see, for example, Juschkewitsch, *Geschichte der Mathematik*; Herrmann, *Mathematik im Mittelalter*; Wußing, *6000 Jahre Mathematik* 1.

stands in juxtaposition to the first one: There are, on the one hand, the theoretical texts like the Sacrobosco »Algorism« or the introductions to the different »species«[7] in the reckoning books that use hardly any examples and rely on a descriptive approach to the topic. On the other hand, we find highly practical texts that are explicitly interested in the »how-to« aspect of practical arithmetic, explaining steps of calculations not only with words but also giving lengthy examples and sample calculations. What we find is a highly heterogeneous collection of texts that contains information on the development of late medieval mathematics and of a German mathematical jargon, but also on late medieval history and culture.

To get a better understanding of what we can expect from our quite large research corpus, we decided to examine the 10 relevant manuscripts held by the Austrian National Library more closely. The goal was to determine whether our research focus was adequate, our questions on the material answerable, and whether we might have overlooked problems that need attention. Furthermore, working with a small corpus at first will give us information on what to expect from the process of manuscript description as well as the transcription process in terms of necessary attention to detail and the associated time management. This paper will give information on the exemplary corpus and our connected questions. We will firstly give a list of the manuscripts in question, adding some basic information on when the manuscripts were written/compiled and their content. In a second step we will test our mode of manuscript description on our corpus, focusing not on a detailed catalog entry but rather on an inclusive description of the arithmetic content that uses extant catalog entries and existing descriptions to give the needed paleographical and codicological information. Thirdly, we will concentrate on the challenges of transcribing arithmetic manuscripts. For this project we are using the transcription software Transkribus, which enables us to use its handwritten text recognition (HTR) tool and to train HTR models that explicitly cater to our needs (mathematical content of the Late Middle Ages). In this paper we will discuss the process of creating and improving such a model based on our exemplary corpus. We consider the description and the transcription of the sources as two necessary preliminary steps to ultimately focus on the fourth step for this paper: the analysis of the sources regarding historical, cultural and social questions as well as by language. There are four bigger topics that need to be tackled using our sources: we will look at the context in which the arithmetic texts are embedded, the problems of translation from Latin to German, the dichotomy of theoretical and practical arithmetic and their portrayal in the sources, and the use of mnemotechnical tools to learn and remember the Hindu-Arabic numerals and the positional notation system.

7 »Species« is the name given to the basic arithmetical operations, of which medieval scholars distinguished 9 – in contrast to our 4 today (addition, subtraction, multiplication and division): *Numeratio*, *Additio*, *Subtractio*, *Multiplicatio*, *Divisio*, *Mediatio*, *Duplatio*, *Progressio*, *Radicum Extractio*; see, for example, Schirmer, *Wortschatz der Mathematik*, 67-68.

The Late Medieval and Early Modern Arithmetic Manuscripts of The Austrian National Library Written in German.

The ten manuscripts we examined at the Austrian National Library are all dated to the 15th and 16th centuries. Five of those manuscripts have already been digitized (Cod. 3029, Cod. 5206, Cod. 5277, Cod. 3528 and Cod. 5184), whereas the other half of the corpus (Cod. 10788, Cod. 10753, Cod. 10939, Cod. 2976 and Cod. 3502) needed to be examined in more detail to determine whether the manuscripts were relevant for the research project. Three manuscripts, Cod. 10753, Cod. 10939 and Cod. 10788 were written in the later 16th century and therefore after the cut-off date we set for this research project. What is interesting about these manuscripts, though, is the fact that they are not built like »classical« reckoning books. They are very obviously made for traders and everyday use, but they neither include theoretical and practical information on the nine species, nor do they give an abundance of reckoning examples as other reckoning books from the 15th and early 16th century do. They are rather built as reference books. They contain mainly conversion tables for currencies and weights of different German and even European cities. The books are utilizing the skills taught by reckoning masters but do not fall into the category of a reckoning book, even though conversion tables and information on currencies and weights are integral parts of earlier reckoning books as well. The difference is that in a reckoning book that functions as an instructive manual, the economic information is embedded into the educational objective of teaching the art of arithmetic with pen and paper. What seems to have happened is that over time the instruction in theoretical and practical arithmetic was detached from the everyday application of the learned skills as a support for the working merchant. What the exemplary corpus of the Austrian National Library shows is the development of a handwritten tradition that builds on the knowledge taught by reckoning masters and that does not need to dwell on the teaching of arithmetic skills anymore. Being able to read and use the Hindu-Arabic numerals is the prerequisite for writing and utilizing books like Cod. 10753, Cod. 10939 and Cod. 10788, which indicates that in the second half of the 16th century the positional notation system and the cultural technique of calculating with pen and paper have fully arrived in the vernacular society.[8]

The other seven manuscripts we studied in more detail were all incorporated into our research corpus. More information on the codices in question is given below in chronological order, starting with the oldest manuscript of our exemplary corpus:[9]

8 This observation is only valid for the area of southern Germany. The expansion of the Hindu-Arabic numerals is different throughout Europe: Italy had already developed reckoning schools and vernacular textbooks in the 14th century and German traders sent their children to Italy to learn the positional notation system long before the first reckoning schools were established in Germany in around 1500. In northern Germany, on the other hand, we have no evidence of the use of Hindu-Arabic numerals before the second half of the 16th century. The dispersion of knowledge on the Hindu-Arabic numerals started in the south and slowly made its way up north. The expansion of this knowledge and the connected practices is linked to a rise in economic activity due to the easier method of calculation. Cf. Danna, *et al.*, Numerical revolution.

9 The information on the following manuscripts was gathered using the existing catalogs and databases that are further described and used in the section »Description of Manuscripts«, namely in footnote 15 in detail. The main two sources are: Menhardt, *Verzeichnis* and *Tabulae codicum manu scriptorum*, ed. Academia Caesarea, vol. II: Cod. 2001-3500.

Other literature consulted: Curtze, Eine Studienreise, at 287 and fn. 61; *Algorismus Ratisbonensis*, ed. Vogel; Reiner, *Terminologie der ältesten mathematischen Werke*, 17; Brey, Deutsche mathematische Texte des 15. Jahrhunderts; Glaßner, *Katalog der deutschen Handschriften*, 640; Wiesinger, ›Als der algoriſmus ſpricht ...‹, 7-25.

Codex 3029 is the oldest of the ten examined manuscripts and was written in the first half of he 15th century; Hermann Menhardt loosely dates the codex between 1425 and 1452. We are looking at a very early German reckoning book that directly caters to readers who must use arithmetic on a very practical level – merchants or craftsmen – and gives information and examples on how to calculate with integers and fractions. This manuscript is a stand-alone reckoning book that is not bound together with any other texts and was apparently used to inform the readers on the practical use of the Hindu-Arabic numerals in detail. Codex 3029 is closely connected to the Bamberger Rechenbuch, printed in 1483 in Nürnberg.[10] Both texts are linked to the *Algorismus Ratisbonensis*, a very early (1st half of the 15th century) reckoning book written in St. Emmeram in a mixture of German and Latin that can be traced back to Fibonacci's *Liber Abaci* and the European beginnings of the reckoning book tradition.

The next oldest manuscript is Codex 3502, a miscellany which can be dated to the second half of the 15th century. It combines a variety of texts in Latin and German covering topics on grammar and lexicology, rhetoric, the art of writing letters, treatises on the computus, the calendar, and of course, several texts on the art of arithmetic. One of them is one of the seven known German translations of the introduction to the art of reckoning with Hindu-Arabic numerals, better known as the »Algorism«, by Johannes de Sacrobsco[11] on folio 270r-277r. A Latin version of the Sacrobosco »Algorism«[12] overloaded with Latin glosses can be found in the same manuscript on folios 148r-167v; two more algorism treatises, not following the Sacrobsco tradition, can also be found in the miscellany. More German mathematical text fragments and reckoning examples are distributed incoherently throughout the whole manuscript. One double page (folios 147v and 148r) contains very dense information on how to do calculations using the Hindu-Arabic numerals and with a counting frame. The analysis of the watermarks shows that the parts of the miscellany that contain German arithmetic texts can be dated between 1474 and 1488.

10 Rath, Über ein deutsches Rechenbuch.

11 The original title of this treatise was »De arte numerandi«, the art of numeracy, written between 1225 and 1230. It was the first treatise on the Hindu-Arabic numerals that got introduced into the university curriculum, where it was studied well into the 16th century as the basic information on the art of arithmetic (Smith and Karpinsky, *Hindu-Arabic Numerals*, 58-59). F. Saaby Pedersen attempted to edit this text mainly based on the Copenhagen manuscript (København, Kgl. Bibl., Ny kgl. Saml. 275^{a},4°. Vellum, 20 ½ x 14 ½, ff.85, Iate 13th c.) and additionally on both the Oxford and the Stockholm manuscripts; *Petri Philomenae de Dacia et Petri de S. Audomaro opera quadrivialia*, ed. Pedersen, 174-201). An edition using all extant sources seems almost impossible given the ubiquity of the source material. Pedersen argues that there might be more than 50 »Algorisms« in Latin; information from the Jordanus database, which was established roughly 10 years after Pedersen published his edition, leads us to believe, that we are dealing with a far bigger corpus with hundreds of extant manuscripts. The name »Algorism« stems from a malapropism of the Arabic name Al-Khwarizmi and soon became the expression under which the general introduction to the Hindu-Arabic numerals was known.

12 The algorism texts that follow Sacroboscos' treatise can easily be identified via the incipit that reads *Omnia, quae a primaeva rerum origine processerunt* [...] (cf. *Petri Philomenae de Dacia et Petri de S. Audomaro opera quadrivialia*, ed. Pedersen, 174).

Cod. 5206 was written in the second half of the 15th century in Latin and for the most part in German. It contains medical, alchemical, and astronomical treatises as well as a treatise on the art of memory, recipes, and roughly 20 pages of reckoning examples in German (folios 81r-94v). On folio 39r a German reckoning example was added to a Latin text on arithmetic (folios 36r-41r) and can almost be considered as a form of commentary because the vernacular example is given in accordance with the content of the Latin text and thematically connected to four passages on the page via a token. The miscellany Cod. 3528 was compiled over the course of 60 years between 1480 and 1540. It contains, among other things, historical, astronomical and astrological treatises in Latin and German, including a computus and rules for good health in the latter language. Even though there are no mathematical texts in Latin handed down in this miscellany, we can identify two German mathematical texts that are connected to the realm of trade: The first is an instruction on how to make a measuring rod accompanied by several illustrations. Those texts known as »Visiertraktate« in German are part of the geometrical tradition and transmit very practical knowledge.[13] The second mathematical text is a compilation of reckoning examples on folios 205r to 210r. They mainly cover the rule of three, exemplified by trade problems and adorned by small illustrations that refer to the content of the corresponding reckoning example.

The manuscript with the shelf mark Cod. 2976 contains treatises from 1481 and later years. It is again a bilingual (Latin and German) miscellany that contains texts on several different topics including astronomy, rules for good health, information on the weather, the black death, and several treatises on the calendar. Most of the manuscript is written in German, including the arithmetic treatise, which is a reckoning book (*Nürnberger Rechenbuch*) on how to calculate with a counting frame (»Linienrechnen«, which means reckoning on the lines [of the abacus]).

Cod. 5184 can be dated to 1485 and only contains one short piece of text that has a loose connection to arithmetic: On folio 60v we can find a mnemonic on the shape of the Hindu-Arabic numerals known as »vnvm dat vinger«. It is a bilingual mnemonic that helps to memorize the shape of the numbers via everyday items of similar form. Other German parts of the miscellany deal with astronomy and medicine; a large part of the manuscript is written in Latin and mainly deals with astronomical problems.

Codex 5277 was written in the first half of the 16th century[14] and can be considered a mathematical manuscript mainly written in Latin. It contains information on arithmetic, geometry, and algebra; German parts are embedded in the first text, a treatise on algebra which is regularly interrupted by German reckoning examples that deal with arithmetic and algebraic problems. Those are inserted by the same hand that wrote the Latin treatise.

13 Folkerts, *Die Faßmessung (Visierkunst).*

14 Two exact dates (2 November 1520 and 25 October 1525) can be found in the manuscript on the folios 100v and 236v (cf. Unterkircher *et al.*, *Datierte Handschriften in Wien*, 52-53).

Description of Manuscripts

First Considerations

These seven ÖNB manuscripts serve as an exemplary corpus to undertake initial considerations about how all our manuscripts can be described. A classical catalog is out of the question for two reasons: firstly, because the textual witnesses are scattered throughout the entire German-speaking area and beyond, making it basically impossible to examine all of them, and secondly, because the focus of the project – namely the edition of the German-language arithmetical components of these manuscripts – does not require in-depth indexing. Therefore, the first step will be to produce short descriptions based on existing descriptive sources which can then be supplemented by individual observations that result from personal inspection of manuscripts or from findings that can be gathered from digital copies. A strong focus of the description will be on the arithmetic parts/treatises of the manuscripts.

A fundamental problem with this approach is the disparate and unfortunately often deficient existing descriptions of the individual manuscripts. In particular, paper codices from the late Middle Ages, which are rarely illuminated and in many cases constitute complex miscellanies, are usually only poorly cataloged as many cataloging projects proceed chronologically or thematically and do not include the group just described. This problem also affects the seven manuscripts of the Austrian National Library that are relevant to our project. They are recorded in the following catalogs/databases: [15]

Shelf mark	ONB online	Tabulae	Dat. Hss.	Menhardt	manu-scripta	HS-Census	Jordanus
Cod. 2976	x	x	x	x	x	x	x
Cod. 3029	x	x		x	x	x	x
Cod. 3502	x	x	x	x	x	x	x
Cod. 3528	x	x	x	x	x	x	x
Cod. 5206	x	x	x	x	x	x	x
Cod. 5277	x	x	x		x		x
Cod. 5184	x	x	x	x	x	x	x

All seven codices can be accessed via the online catalog of the ÖNB.[16] The information on these manuscripts listed there was taken from the *Tabulae codicum manu scriptorum*[17] from 1886, which at least provides basic data. The entries on *manuscripta.at* also follow the *Tabulae*. Although the texts are all named, they are not identified according to today's standards. Foliation, *inicia* and many codicological details are missing. Six of the seven codices contain datings and are thus included in the *Catalogue of Dated Manuscripts* by Franz Unterkirchner,

15 www.onb.ac.at/bibliothek/sammlungen; *Tabulae codicum manu scriptorum*, ed. Academia Caesarea; Unterkircher *et al.*, *Datierte Handschriften in Wien*; Menhardt, *Verzeichnis*; manuscripta.at; handschriftencensus.de; ptolemaeus.badw.de/jordanus/start.

16 www.onb.ac.at/ (accessed on 13 March 2023).

17 *Tabulae codicum manu scriptorum*, ed. Academia Caesarea, vol. II: Cod. 2001-3500.

where they were summarily described and depicted. For those five manuscripts where the *Tabulae* list German-language texts, we have entries in Menhardt's *Catalog of German Manuscripts* as well as articles in the Handschriftencensus – both, however, concentrate on the German-language entries and disregard the Latin ones. There is further literature on some of the codices, but it is devoted to the contents of individual texts, not to the manuscripts.

For a repertory within the ARITHMETIC online edition, a very fundamental decision must therefore be made right at the beginning: is it legitimate to generate entries of widely varying detail, or is it better to limit oneself to a few basic data (such as place of storage, material, size, dating, summary of contents, or the like), which must be compulsorily collected for all manuscripts and are thus valid? The Jordanus database, an initiated index of all vernacular mathematical texts, in which all seven ÖNB manuscripts appear, has chosen the latter path. The entries there differ in length and quality but make their sources transparent and traceable. However, since the database has not been hosted for years, the entries, some of which have also not been edited, are unfortunately not a reliable basis. Nevertheless, this approach – bringing together the relevant information from different sources – seems like a sensible decision. After all, omitting already existing information just to ensure conformity of the catalog entries would be wasteful. In either case it will be important to clearly identify the sources and to contextualize them to enable the recipients of the database to classify them. But this approach also raises new questions. First, the »relevant« information mentioned above. What is this in the context of the ARITHMETIC project, which not only wants to generate a list of manuscripts but also to interpret the arithmetical texts in a multi-perspective context?

Basic Data and Non-Arithmetical Content

The focus of the descriptions is on the content of the manuscripts, but how is this to be reproduced in the repertory? Most textual witnesses are miscellanies that contain many texts. And these do not all deal with arithmetic and are not all German. An exact determination and description of all the texts contained would mean an enormous consumption of time and is not of primary importance for the question of the arithmetic texts. Nevertheless, the surrounding texts are significant when it comes to knowledge systems, reading matter, usage, or possible recipients of the mathematical components.

Due to this initial situation, the project team has decided to keep the descriptions of the manuscripts short, to collect the existing basic data in a headline (shelf mark, place of storage, dating, size, material, extent) and to supplement them if necessary. This headline should be followed by the fields »Provenance« and »History/Owners«. Here, data on the origin, dialect, previous owners, olim shelf marks, etc. can be collected. A »Characteristics« field gathers information on the textual environment and places the entire manuscript in a larger context (e.g., university, monastery, trade, etc.) based on the combined codicological and content-related findings. After these short descriptions, references to databases and literature are given. The last field is devoted to special features concerning the writing in the German-language arithmetical texts. Here, deviations from the general edition guidelines are noted individually for each manuscript.[18]

18 At this point in time within the project duration, it is not yet possible to determine the exact way in which the German-language arithmetic texts will be included in the repository. These decisions will be made when the corpus has been reviewed and analyzed in its entirety.

Draft of a description for the repository:

Wien, Österreichische Nationalbibliothek Cod. 3502

2nd half of the 15th century (1472-77, with supplements until 1499), paper, 220×155 mm, III+278 folios

Provenance:	Ingolstadt (?); Upper Franconian dialect (Menhardt)
History/Owners:	Johannes Franciscus ex Eschenbach (fol. IIIv): scholar in Gmünd, 1472-1472 student in Ingolstadt (Unterkircher) Wolfgangus Plessinger ex Eschenbach (fol. IIIv, 278r; c. 1500). (Unterkircher) Vienna, Old University Library (after 1623 - until 1756): Olim signature *Univ. 496*.
Characteristic:	Predominantly Latin miscellany with grammar, rhetoric, astronomy, a computus, mnemonics, a Latin algorism as well as a love letter from a student to a bourgeois woman by Samuel Karoch von Lichtenberg. Not only through its previous owners, but also through this text (the author himself was a magister at the newly founded university in 1472; see DB), the manuscript shows a strong connection to the university in Ingoldstadt. The codicological structure of the manuscript, consisting of several bound fascicles with individual texts written by 12 scribes, is also appropriate for a university manuscript. The manuscript is rubricated throughout, but is otherwise free of decoration.
Databases:	https://manuscripta.at/hs_detail.php?ID=6954 https://handschriftencensus.de/11585 https://ptolemaeus.badw.de/jordanus/ms/11745
Literature:	Hermann Menhardt, Verzeichnis der altdeutschen literarischen Handschriften der Österreichischen Nationalbibliothek. Bd. 2 (Veröffentlichungen des Instituts für deutsche Sprache und Literatur [der] Deutsche[n] Akademie der Wissenschaften zu Berlin 13). Berlin 1961. […]
Notes on the edition:	[…]

The non-arithmetic texts will be described only summarily in a first step, focusing on their general fields of knowledge (e.g., astronomy, medicine, geometry, philosophy, etc.). Even if these data are only summarily recorded, they still have significance, for it makes a difference whether a manuscript »only« transmits a reckoning book with a strong reference to trade, or whether the mathematical contents stand in the context of computistics or perhaps academic scholarship.[19] If towards the end of the project time allows, the transmitted texts will be defined more precisely by indicating the *inicia* or even by being identified.

19 Further considerations on the context of transmission are elaborated in the section »German Arithmetic Manuscripts of the Austrian National Library: Texts and Context«.

The History of Ownership and the Context of Transmission
Using an example from the manuscripts examined in the ÖNB, the relevance of the history of ownership for our project can be illustrated:

Cod. 5206 is a scientific manuscript compiled from the second half of the 15th century to the beginning of the 16th century.[20] It contains 179 folia of recipes, memorial art, astronomical, mantic, medical and mathematical texts, most of which are in German. The composition of the texts is remotely reminiscent of a so-called house book. House books belong to the nobility or bourgeoisie, and that is where one would look for the provenance of this miscellany. Although the manuscript itself has no written evidence of its previous owners, it can nevertheless be assigned. Together with Cod. 5277, it reached the then Wiener Hofbibliothek in 1655, when the descendants of Georg and his son Philipp Eduard Fugger sold about 300 volumes of their large collection to the Austrian emperor.[21] Part of the sold Fugger library was also the majority of the book collection of Johannes Schöner, a Nuremberg geographer, astronomer, mathematician and clergyman,[22] who not only collected scientific literature but also copied it personally, published it as a printer, and even wrote it himself.[23] But he had also drawn from another collection: his library contained several books from the estate of Johannes Regiomontanus, which included the latter's own works (partly as autographs) but also texts by Georg von Peuerbach.[24]

This single manuscript alone makes it clear that German-language arithmetical texts cannot be considered without their context of transmission. The ownership history of Cod. 5206 reveals an entire network of scholars in which mathematical texts circulated. However, it also shows the textual environments in which they can be found, and how the composition of the content can provide clues to their owners. Therefore, it will be important to highlight this information when describing the manuscripts. One way of doing this would be to include a text that provides a brief characterization of the manuscript in addition to the collection of basic data and the textual environment. Here, reflections on the context of transmission within the manuscript, but also within the collection from which it comes, could be included and the provenance history, if it can be reconstructed, could be discussed.

20 Cf. Menhardt, *Verzeichnis*, 1110.
21 Franz, Handschriften aus dem Besitz des Philipp Eduard Fugger, 61-62.
22 www.deutsche-digitale-bibliothek.de/person/gnd/118795341.
23 Maruska and Schöner, *»Homo est nescio qualis«*, 113-150.
24 Maruska and Schöner, *»Homo est nescio qualis«*, 78.

Process of Transcription

In this paper, we have already discussed the heterogeneity of our whole and also our exemplary corpus in detail. The texts vary between theoretical treatises with comparatively few reckoning examples or tables and whole reckoning books with often more than 100 folios full of practical mathematical knowledge and the corresponding reckoning examples and calculations. This lack of uniformity in our sources poses a problem when it comes to the process of editing and transcribing them. Editing and transcription techniques that have been established and improved over the last two decades and can be considered well-known by now[25] can be used to edit theoretical treatises that are usually constructed as »simple« text and only rarely use calculation, tables, or diagrams. Our problems begin when we are confronted with more than just running text: the embedding of calculations, figures, tables, glosses and other forms of paratext poses a difficulty that calls for custom solutions, which we will develop in collaboration with the Center for Information Modelling based at the University of Graz.[26] Our small corpus from the Austrian National Library can be considered to be the perfect example for the problems we are dealing with, which we will elaborate in more detail below.

25 On the benefits and implementation of text-based transcriptions and (digital) editions in the field of German studies and history, see, for example, Bein, Editionsprinzipien für deutsche Texte; Thumser, Verfahrensweisen; Huygens (ed.), *Ars edendi. A practical introduction*; Nott-Kofoth, Editionswissenschaft; Sahle, *Digitale Editionsformen 1*; Bleier *et al.*, Digitale Mediävistik. But also guidelines for transcription and edition should be mentioned, such as Empfehlungen zur Edition frühneuzeitlicher Texte des Arbeitskreises »Editionsprobleme der Frühen Neuzeit« (AHF), accessed on 28 March 2023: www.heimatforschung-regensburg.de/280/ or Transkriptionsrichtlinie Archivschule Marburg (Mittelalter/Frühe Neuzeit), accessed on 28 March 2023: www.archivschule.de/DE/ausbildung/transkriptionsrichtlinie/.

26 The ZIM (Zentrum für Informationsmodellierung – Austrian Center of Digital Humanities) located at the University of Graz (informationsmodellierung.uni-graz.at/de/) and led by Georg Vogeler (online.uni-graz.at/kfu_online/wbForschungsportal.cbShowPortal?pPersonNr=80075) focuses on the research field of digital editions, which are understood as a generally applicable humanities method for the semantic and formal indexing of cultural artefacts. Long-term archiving is also a major interest of ZIM's research fields. The planned digital edition of the project is to be published via GAMS (Geisteswissenschaftliches Asset Management System), which is managed by ZIM. GAMS is an OAIS (Open Archival Information System)-compliant asset management system for the management, publication and long-term archiving of digital resources from the humanities (gams.uni-graz.at/context:gams?locale=en). A project which ARITHMETIC is methodically oriented towards is CoReMA (Cooking Recipes of the Middle Ages), supervised by GAMS (gams.uni-graz.at/context:corema).

Our research corpus, consisting of almost 140 manuscripts, is quite substantial. This means that we need HTR support to transcribe and subsequently digitally edit all the relevant texts. We have chosen to work with Transkribus[27] because after years of intensive use by scholars and regular updates, it is not only user friendly and offers many helpful tools but it also provides a support team we can turn to. Furthermore, the process of semantic enrichment can already be started in Transkribus and allows for an easy conversion into a TEI.[28] The program offers an extensive selection of existing tags that can be used for further processing, but there is the possibility of creating customized tags as well. While working with Transkribus on our exemplary corpus we were able to observe that our problems with the software are due to the analysis of complicated layouts and the unsuitability of publicly available HTR models for most of our primary sources. Therefore, we made the choice to train our own HTR models[29] to support the transcription process, which we will describe further below. Another issue we observed while dealing with our exemplary corpus is that Transkribus' strong focus on (running) scripture requires specific model training to allow for a somewhat successful recognition of mathematical parts, i.e., numerals or fractions.

To facilitate a wide use of our transcription for various disciplines we decided to stick to the handwritten text as closely as possible during the transcription process. This means that we do not interfere with the scribe's language, and we do not change the syntax. We also do not interfere with grammar or punctuation; upper- and lower-case letters are adopted from the text, regardless of syntax; and the scribe's characters are adopted, as well as every letter form used in the texts: long S (ſ), the Z forms (z, ʒ,); and neither I or J nor U or V will be normalized. Superscripts are also adopted. Interventions are only made when it comes to the i-dots, which are systematically set; the y-dots (ÿ) are not depicted in the transcription because they are not used consistently and do not have any phonetic meaning.

27 Transkribus (readcoop.eu/transkribus/) is a text recognition software whose primary goal is to support work with historical documents. Originally, the software was developed within the framework of an EU FP7 project and subsequently developed further within an EU project »READ« (Recognition and Enrichment of Archival Documents) led by the University of Innsbruck (www.uibk.ac.at/de/) in 2020. Now it is being continued by a European Cooperative Society (SCE). The AI provides recognition of text, layout and structure. Furthermore, it is possible to train the AI by yourself, i.e., to use your own material for text and layout recognition training. For a systematic review of the platform, see Nockels *et al.*, Understanding the application of handwritten technology, accessed on 29 March 2023: doi.org/10.1007/s10502-022-09397-0.

28 TEI (Text Encoding Initiative) is a set of rules on which manual annotation in the XML (eXtensible Markup Language) editor Oxygen is based. The TEI consists of an XML-based markup language that humanities disciplines have been using since the early 2000s as a tool for digital annotation and description of texts. See, for example, Sahle, *Digitale Editionsformen 3*, or Driscoll and Pierazzo (eds.), *Digital Scholarly Editing*. See the guidelines for TEI at www.tei-c.org (accessed on 11 January 2023).

29 Although model training in Transkribus is very uncomplicated, some disadvantages should be pointed out: First, the trained model cannot be exported from Transkribus. Thus, the model will be lost if something should happen to the software. Second, Transkribus charges an average of 1 credit for reading a page with the AI (you get 500 credits for free after initial registration). Once the credits have been used up, you have to buy more from the platform, so it is not completely free of charge.

The Transcription Process: First Experiences

Our initial approach to the process of handwritten text recognition was to use one of the publicly available HTR models for the recognition of 15th-century German handwritings provided by Transkribus and to test it on our exemplary corpus. We quickly encountered problems with the recognition of various symbols and graphics, but also (and especially) with mathematical figures such as numbers, fractions etc. In terms of recognizing written text besides numbers and mathematical symbols, the public model achieved viable results for Codex 3029 but could hardly read the German Sacrobosco »Algorism« in Codex 3502, even though both texts were written in the middle of the 15th century. This led to the decision to use our exemplary corpus to start the process of working on a customized HTR model that supports the recognition of numbers and mathematical symbols a little better than the publicly available model.

Another first observation we made while working on our exemplary corpus concerned layout problems of arithmetical manuscripts and the impact of these on the transcription process: As mentioned before, HTR models work well when they are confronted with continuous text and do not have to read mathematical figures and numbers. In our corpus the theoretical treatises like the Sacrobosco »Algorism« (Cod. 3502, figure 1) fall into that category; these texts explain the arithmetical problems on a very abstract level, using hardly any numbers and not giving sample calculations. The scribe follows a predefined set of lines and sticks to them, with only some exceptions: sometimes we are confronted with additions, corrections, or annotations in the margins, but these do not pose any great difficulties for further processing in Transkribus or later in the XML editor. These texts work like any other medieval treatise usually edited following the guidelines of TEI.

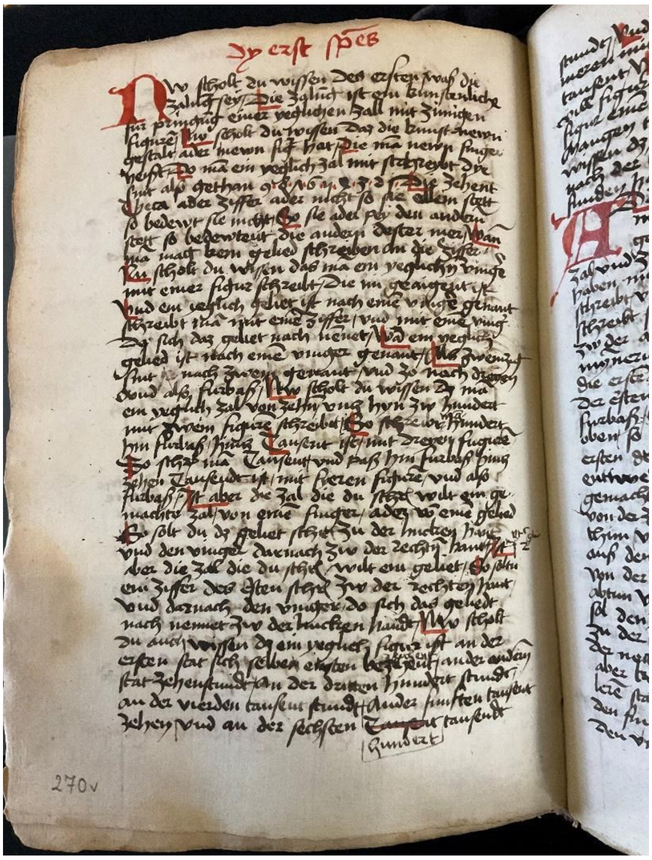

Figure 1: Vienna, Austrian National Library, Cod. 3502

Our problems start with the texts that are concerned with practical arithmetic. Those usually consist of an introduction with a few lines of theory on an arithmetic problem, followed by practical examples, often in the form of conversions, tables, or text problems. Codex 3502 not only includes the Sacrobosco »Algorism« but also contains information on practical arithmetic spread over several folios. Alongside very short snippets on arithmetical rules, we can find reckoning examples, sample calculations, and conversion tables. Folio 147v in particular (see figure 2) needs to be mentioned here.

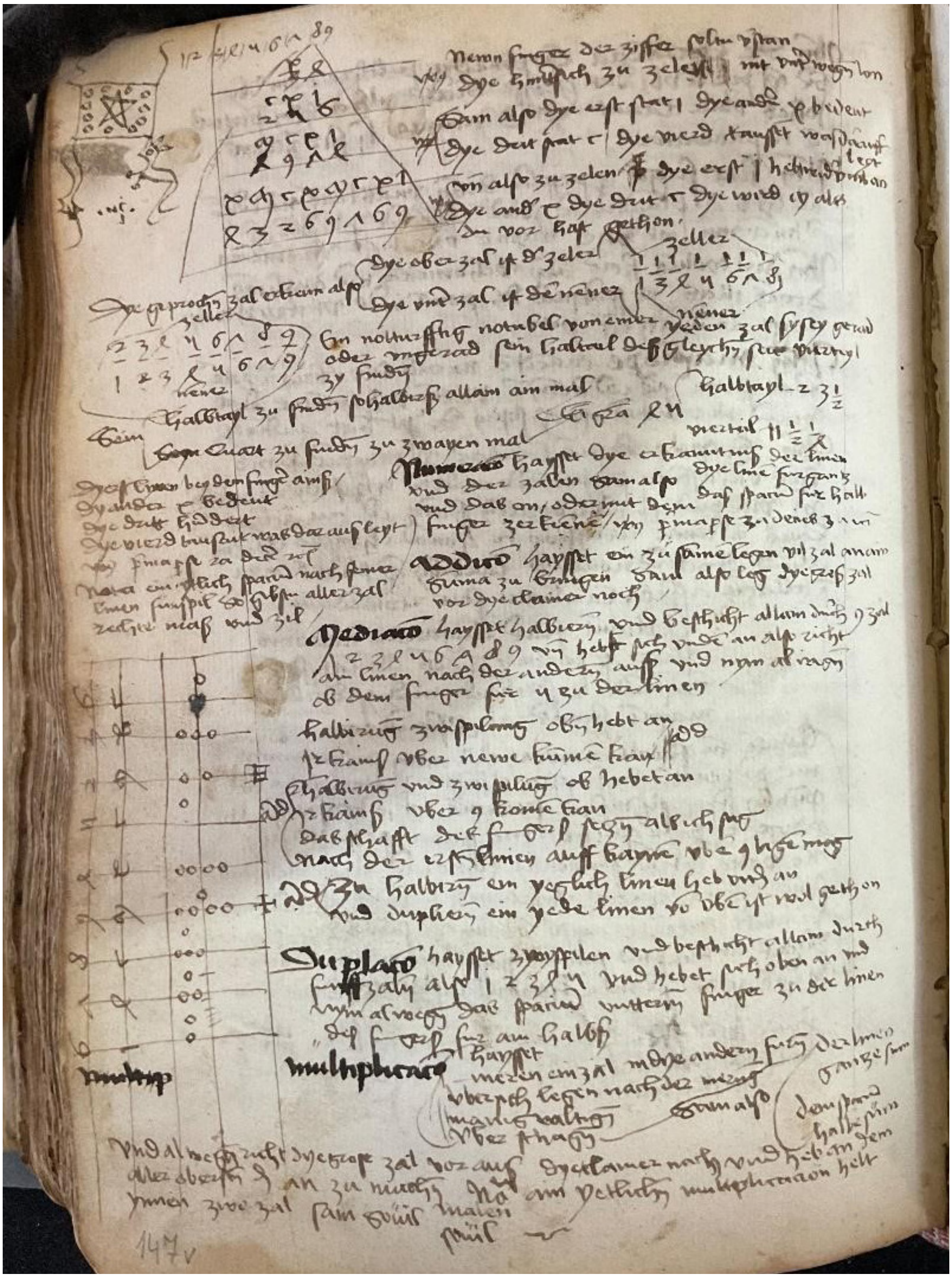

Figure 2: Vienna, Austrian National Library, Cod. 3502, fol. 147v

This extraordinary layout is exemplary for the various difficulties we will be dealing with during this project. In contrast to Figure 1, the scribe does not follow any structure. The order of the lines cannot be determined with certainty and the scribe uses a series of cross-references that are either connected to figural/symbolic depictions or to other text passages which do not necessarily have to be on the same page. Between the very short descriptions of the individual species on folio 147v (*numeracio*, *addicio*, *mediacio*, *duplacio*, *multiplicacio*),[30] we can see rhyming explanations on fractions, which presumably enhance memorization.[31] Right next to that, line calculations can be traced on the abacus.

A pyramid of numbers with Roman and Hindu-Arabic numerals can be found in the upper left margin of the page and an illustration was inserted next to it, possibly depicting a fanciful coat of arms. Although this gives the impression that these pages have no structure, exactly the opposite is the case. Using only two pages, the writer summarizes everything a person needs to know about numbers: the abacus and the species of *numeratio* with Hindu-Arabic numerals are introduced; fractions are explained; the most important species are summarized; and conversions of measures, weights and currencies are offered – in short, everything a merchant in the 15th century needed to know to carry out his business daily. A solution for how to transcribe and digitally edit this sheet still needs to be developed. Problems are not only caused by notational issues, such as the introduction of fractions or the depiction of line calculations on the abacus, but also by cross-references that can refer to content on other pages. An example for that is the pyramid on folio 147v. Next to this figure we can see three symbols of the same type followed by a text connected to the pyramid. These little symbols can also be found on folio 148r and serve as references that connect content from both pages. However, Transkribus unfortunately does not offer the technical possibility to transcribe cross-references over more than one page, which can pose a problem for the transcription of some texts, including several from our corpus.

Another problem we have encountered with the software is connected to the layout analysis Transkribus offers. We can define certain text fields by the content they contain, but we cannot further process or even transcribe the data. For example, Transkribus offers a specific text field named »Maths«. If we identify mathematical operations via this text field and start the layout recognition, the field is not considered and is ignored. The situation is very similar when it comes to tables. We can find tables in many manuscripts throughout our whole corpus and need to develop a transcription process to deal with the different shapes and forms of tables we find in arithmetical texts. The simplest form of a table, such as a multiplication table realized as a simple grid holding numbers in its cells, should not cause too much trouble in the editing process since Transkribus offers a table function explicitly trying to ease the process of transcribing such »classic« tables. But we can also find tables that predominantly contain text or are structured like tables but lack the grid that defines the table. Conversion tables fall into that category and can be quite difficult to handle. For those cases, mathematical operations and figures, representations of line reckoning, and tables, we are working on necessary individual solutions with the ZIM.

30 The species *diuisio* is added on fol. 175r in the manuscript.

31 For example: *dye grosser zal am anfang ausricht / dye clainern darnach auch bedicht.* Vienna, Austrian National Library, Cod. 3502, fol. 148r.

Another problem we have encountered that is loosely linked to the layout issues mentioned above is the special characters we are confronted with in historic arithmetic texts: we have various abbreviations, signs for units of measurement, weights, and currencies, and even the development of arithmetic operators and signs for punctuation can be a problem. Some of these difficulties can be shown in Cod. 3029 (figure 3), a stand-alone reckoning book and the oldest manuscript in our Viennese corpus.

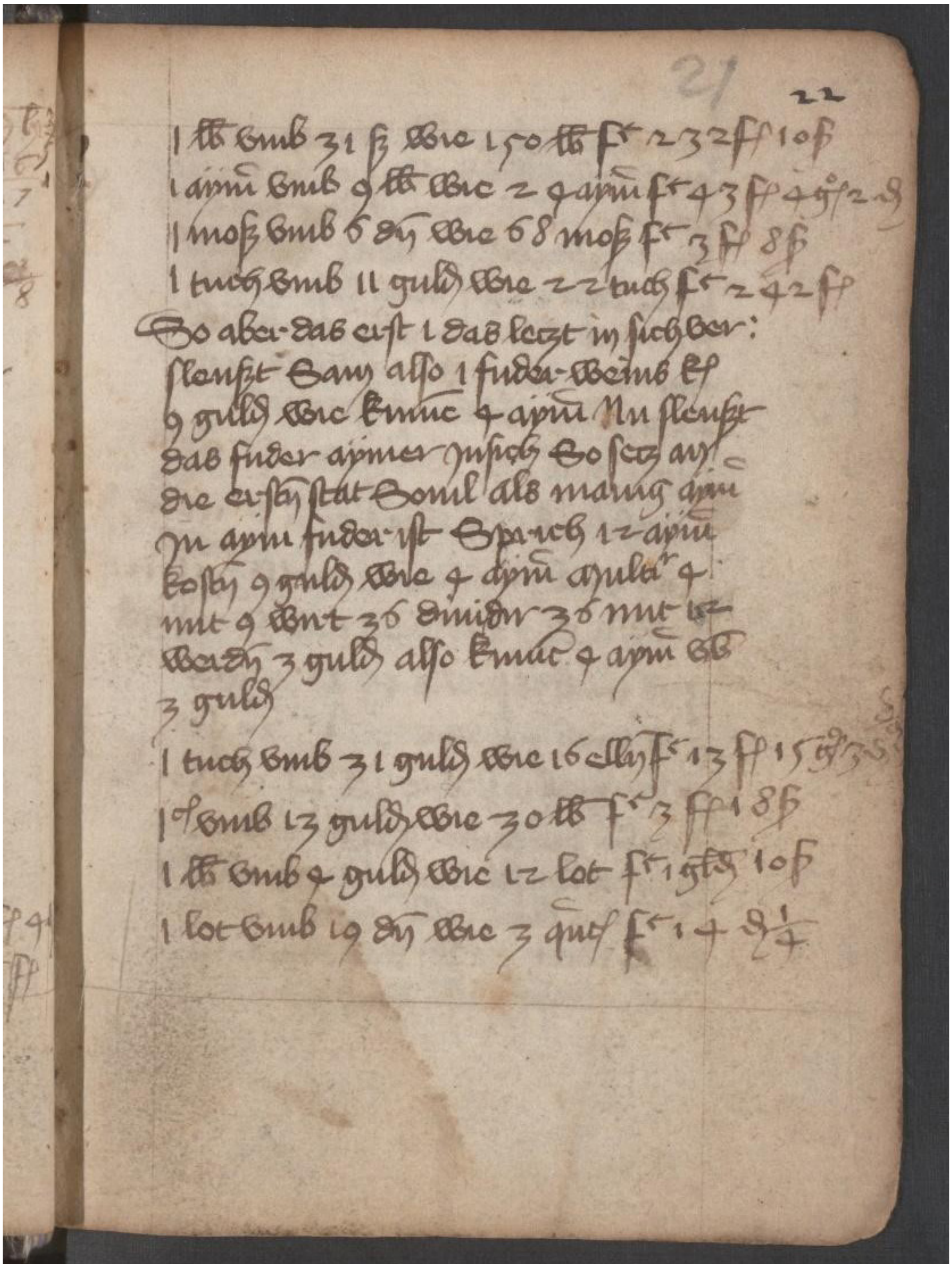

Figure 3: Vienna, Austrian National Library, Cod. 3029, fol. 22r

On folio 22r we can see single letters functioning as abbreviations for whole words, signs for currencies, units of measure as well as abbreviations of those: the abbreviations for pound (ƚƀ), gulden (gulꝺ), denarius (dn̄), solidi or schilling (ſʒ) and groschen (gꝛ̊ſ) can be seen in the first four lines. Even though the abbreviations are mostly stable within one manuscript, variations can and do occur when the corpus is a little bigger. We are even confronted with abbreviations and special characters for units we cannot identify. When it comes to transcribing those special characters and abbreviations, we currently rely on the characters developed by the Medieval Unicode Font Initiative (MUFI).[32] Even though the MUFI develops new characters constantly and those are also regularly integrated into the Transkribus software, we will still need to develop several characters with a graphic designer in collaboration with the ZIM.

HTR Training

Since the public HTR model did not perform well for the manuscripts in our exemplary corpus, we decided to invest more time in creating our own model for the whole project. We started by focusing on the 15th-century manuscripts of our exemplary corpus, building our HTR model around those first codices. The question of a reasonable method for the HTR-model training arose and influenced our decision on what manuscripts to transcribe first. Since a homogeneous corpus, preferably a whole manuscript written by one hand, is recommended for the best training results, we had to adapt our method. We only have a handful of manuscripts dated to the 15th century that fit this description; mostly we are dealing with several leaves of text in a miscellany and creating an HTR model for those kinds of texts requires the inclusion of a wider range of manuscripts. Therefore, we decided to start by partially transcribing five texts written in the 15th century as a basis for the whole model and work on improving our results step by step by including more transcriptions, amending mistakes and correcting base lines.

The very first model was our test model. We knew even before we started the training process that the probability of creating a successful HTR model was very low. We used Codex 3029 from the Austrian National Library as the basis for this test model, and our main goal was to test how machine learning works and how it deals with very few pages that fall massively short of the recommended amount of at least 10,000 perfectly transcribed words for training purposes. The training set[33] consisted of 33 pages and the validation set[34] of only 5 pages. The error rate of the validation set was 11.40%. The model achieved surprisingly good results for this one manuscript, but of course we could not use it for any other manuscripts.

32 MUFI (Medieval Unicode Font Initiative) was founded back in 2001 and now consists of a group of scientists and font designers who have committed themselves to finding solutions for the encoding and display of special characters in medieval texts written in the Latin alphabet; accessed on 30 March 2023: mufi.info/m.php?p=mufi.

33 This set is used by the AI to learn the text, layout and structure of your documents used for the model training.

34 The validation set contains leaves from the same manuscripts used as training set but they are not part of said training set. The trained AI will use this set of leaves to evaluate the performance of the model.

The second model was the one that constitutes the basis for all of our further training. It contained 130 pages in total from four different 15th-century manuscripts. The corpus consisted of two manuscripts from the Austrian National Library, Cod. 3029[35] and Cod. 3502[36]; one manuscript from the Wrocław University Library with the shelf mark B akc. 1948/207[37]; and one manuscript from the State Library Bamberg with the shelf mark Inc. typ. Inc. I 44.[38] This model had an error rate of 9.70% which is not too bad. However, this model was not suitable for further use since we made some rookie mistakes: We did not pay especially close attention to the baselines,[39] but drawing those precisely from the exact beginning to the end of every line is crucial to getting good results from the text recognition process. In addition, it is suggested that you use almost perfect transcriptions for the HTR training, as every mistake in the transcription can falsify the actual result.

As a next step, we accurately proofread our transcriptions, stuck very closely to the recommended guidelines, checked our spelling, and precisely adapted the baselines. After that we trained our first working HTR model. The first and most important thing we noticed with this model was that Transkribus has big problems with recognizing and reading fractions in the running text. They are either not identified as such at all, misread, or the baseline and/or the line region[40] is split in two or completely out of place. To guarantee better accuracy for the overall text recognition, we decided to set the fractions to *unclear*, which omits the fraction from the training. Our corpus was a little different in this try compared to the last training attempt due to some difficulties we had with the digital copies of the Wrocław manuscript. We had to exchange this text for a reckoning book from the Canton Library Aargau with the shelf mark Ms. ZQ 27[41]. A total number of 123 pages were used as our training set and 31 pages were in the validation set for this model. It was possible to improve the accuracy of the validation set by 2%, to 7.60%.

Since our aim was to train a model that can read fractions more reliably, we thought about what steps would be necessary to achieve this goal. We wanted to include the fractions in this step of the training and therefore avoid complications with the baselines and the line regions. We experimented with enclosing the fractions with the baseline so that in the best possible case the model would recognize the fractions as one symbol. This training set consisted of the same manuscripts plus one addition: we added a manuscript from the Berlin National Library with the shelf mark Ms. Germ. Oct. 375.[42] However, this model did not meet our expectations at all. The fractions were not recognized, and subsequently the automatic text recognition was worse than in the last model. This higher error rate could be caused by

35 Pages edited: 2r-38r; dating: first half of the 15th century.

36 Pages edited: 270r-277r; dating: second half of the 15th century. Watermarks in this miscellany were dated to 1474-1488, see Menhardt, *Verzeichnis,* 2, 914.

37 Pages edited: 3r-10r; dating: 1415.

38 Pages edited: 1r-21r; dating: between 1460 and 1470.

39 The baseline acts as a reference point for the text recognition and characterizes a polyline that runs along the bottom of the text line.

40 Similar to the baseline, the line regions are also inside the text region; however, they do not consist of a polyline, but are polygons that enclose the entire handwritten text in a line.

41 Pages edited: 1r-14v, dating: 15th century.

42 Pages edited: 1r-20v; dating: 15th century.

several things, but we strongly suspect that it is mainly due to the enclosing of the fractions and the newly drawn baselines, which were no longer straight and might have thrown off the general text recognition. In addition to that, the newly added Berlin codex could also influence the overall accuracy. The general result of this training was that including the fractions severely affected the accuracy of our model as this was the first training where they were included and not set to *unclear*. Enclosing the fractions with the baselines did not lead to the desired goal of better text recognition; on the contrary, the training set had an error rate of 4.10% and the validation set of 14.01%, doubling the overall error rate.

After this setback, we trained our last model for the moment. We straightened the baselines and had them run through the fraction lines, no longer enclosing the fractions. We wanted the baseline to sit exactly on the fraction bar with the numerator above and the denominator below the baseline, giving the machine the chance to maybe recognize separate numerals above and below. Furthermore, we added the Wrocław[43] manuscript back into the training set, making for a total of 6 codices, 164 training pages in the set and 41 pages in the validation set. This is now our base model for any further training. Due to diligent work and meticulous adjusting, we were able to reduce the error rate to 7.50% again. This model works very well with other hands that are like the ones in our training set but are not part of the training corpus; it also achieves promising results on manuscripts dated before 1520 overall, which is very reassuring. We are still trying to improve our results on the recognition of fractions by adding pages from different manuscripts that show a high density of fractions. We noticed that over time Transkribus recognizes simple fractions like 1/2, 2/3 or 3/4 quite well through regular training. However, more complicated fractions, even though the recognition process is getting better, still cause problems.

Potential Fields of Research

German Arithmetic Manuscripts of the Austrian National Library: Texts and Context

Our final corpus from the Austrian National Library shows that only one manuscript (Cod. 3029) is a stand-alone book that explicitly and exclusively deals with the teaching of arithmetic in German. The other six sources we have examined are miscellanies that bind various treatises in Latin and German together, with arithmetic being only one of many topics we could identify throughout the manuscripts.

The one stand-alone book can be defined as a traditional handwritten reckoning book that has a strong focus on teaching practical arithmetic. That is done by offering very short snippets of theory when a new basic arithmetic operation is introduced, but the focus of this book is on giving one example after another and showing in detail how to operate with integers and fractions. The reckoning examples mainly revolve around the practical implications of trade, touching among other things on conversions, distribution problems, and most of all, account settlement. The size of the book, a small octavo in format, leads to the conclusion that it could not only be read at a library or in a study but was small enough to be carried around and used for the intended purpose: to learn, study, and practice arithmetic at a time when printed reckoning books were not yet widely available.

43 Pages edited: 3r-7r; dating: 1415.

The other German arithmetic texts are a lot less easy to grasp:[44] we have groups of, or sometimes single reckoning examples attached to Latin treatises, short but very precise explanations on various mathematical topics dispersed throughout a whole manuscript or shortened versions of reckoning books bound together with treatises of heterogeneous content. All of the miscellanies we studied have in common that they contain texts in German and in Latin, often significantly more Latin texts than German passages. When we look at the context in which those German arithmetic treatises are transmitted, we can loosely identify three main groups of content that surround these mathematical texts. Identifying those three groups of transmission can shed light on what other fields of knowledge complemented the mathematical texts and were handed down, read, and studied with them: The first are miscellanies with a focus on quadrivium treatises. The quadrivium consists of geometry, arithmetic, music, and astronomy – and most of the treatises transmitted with arithmetic texts in German are other mathematical treatises in Latin and astronomical texts in either German or Latin. Other mathematical disciplines like geometry or algebra can be found in those miscellanies; it also happens that Latin treatises are interspersed with German text portions, as is the case with Codex 5277, when a Latin algebra treatise is repeatedly interrupted by German reckoning examples, written in the same hand. This context group is what we would intuitively expect to be handed down with German arithmetic texts: Latin templates of the vernacular versions of the arithmetic texts, other functional and instructive texts that use mathematical skills (e.g., astronomy, computus, calendar) or explain them further (e.g. other mathematical disciplines like geometry, surveying or trigonometry), and practical skills that utilize numbers and calculation.

A second group of manuscripts is codices that contain texts on medicine, the conservation of good health and (medicinal) recipes. In terms of content these texts are a little further removed from the quadrivium manuscripts of group one, but we can still argue pretty easily that numbers and medicine are two topics that relate to each other: recipes contain indications of quantity, and medicinal advice correlates with astronomical issues.[45] Therefore it is not surprising to find the odd mathematical text in miscellanies that transmit medicinal information, because knowledge of Hindu-Arabic numerals and of mathematics in general allows for an easier understanding of relevant fields of knowledge. Compared to the first group of transmission, we can observe that far fewer manuscripts can be assigned to group two.

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44 We have looked at the whole research corpus to get an understanding of how arithmetical texts are joined together with other texts; our small exemplary corpus from the Austrian National Library fits right into the observations we were able to make on our big corpus.

45 The doctrine of microcosm and macrocosm helps to show the world as available and transparent and to link the role of the individual with the universe. Microcosm (mundus minor, human being) and macrocosm (mundus maior, universe) thereby stand in a unity and interaction that allow larger structures to be used as models of human order. Finckh, *Minor Mundus Homo*. See also: Riha, Mikrokosmos Mensch.

A third and even smaller group of miscellanies is again even further removed from the original mathematical context, but nevertheless we encounter this distinct group of manuscripts on a regular basis throughout the corpus: miscellanies that, from a modern perspective, contain texts we would attribute to the general field of humanities (the trivium [grammar, rhetoric, dialectic], philosophy, history, and the art of memory). They sometimes transmit small arithmetic treatises or other snippets of mathematical content like mnemonics or short reckoning examples. In those codices the contextual environment is a little different and arithmetic texts can serve a whole other purpose than just to inform about mathematics: There is the case of simply needing to fill blank pages; arithmetic texts in these environments are often addenda that make use of an empty space. We also have the case of the university manuscript where different subjects of instruction are pooled together, with arithmetic being just one of them and far from the most important. The result is a heterogeneous manuscript that shows how eclectic collections of knowledge in the 15th and 16th centuries can be.

What does this mean for the practical use of handwritten arithmetic texts in German? For our exemplary corpus we can see that there is only one stand-alone arithmetic manuscript out of the 7 relevant codices in our research period. When we compare this to our full research corpus, we have a different picture: About a quarter of all the manuscripts in our final research corpus are stand-alone handwritten reckoning books in German. The significantly higher number might be connected to the nature of different collections, mainly because other libraries hold more younger texts, and the number of stand-alone reckoning books increases in the first half of the 16th century, at a time when the need for mathematical education in the vernacular increases and leads to the rise of the reckoning schools and the later flood of printed reckoning books. The stand-alone handwritten manuscripts serve the same purpose as the printed books: they are a tool for learning and mastering the art of calculating and people working in trade are their very specific target audience. They can be considered work equipment, something you need for everyday use to help you do your job well. Stand-alone arithmetic manuscripts like Codex 3029 are always reckoning books, they almost always come in a small format like octavo, could therefore have easily been put in a bag or a pocket, and they also show that they have been used. They transmit practical knowledge, are full of application examples, conversion tables, and often contain supplements and glosses.

The arithmetic texts transmitted in miscellanies paint a different picture when it comes to theories about their intended use. Even though we can still see a strong focus on practical arithmetic, the texts overall serve a different purpose: They can be considered collections of a certain degree of knowledge at a moment in time. That includes the transmission of scholarly content in Latin and its vernacular translation and/or revision in the same codex. The miscellanies of our corpus show that in the 15th and 16th centuries some fields of knowledge needed to be expanded by vernacular versions of relevant texts. In the case of mathematical content, we can also see a conflation of theoretical treatises in Latin and the more practical instructions in German that are handed down in the same manuscript. In such an environment the vernacular mathematical texts often serve as complementary knowledge and are either added as a form of commentary to other texts or can be found as addenda where empty space in manuscripts is filled with potentially useful content.

Translation (Latin – German)

The German arithmetical texts in our corpus cannot be examined without their Latin templates in mind especially when it comes to an analysis of the language and the (instructive) style. The earliest German texts we have are either very much dependent on their Latin templates or even multilingual.[46] There we can see the beginnings of a German arithmetical jargon that depends heavily on Latin vocabulary, loanwords, and creative translations into German. In our exemplary corpus we can observe several mechanisms of transmission from Latin to German and the subsequent development of an arithmetical jargon in the vernacular. We want to exemplify that a little further by focusing on one specific manuscript:

The oldest miscellany of our corpus, Codex 3502, hands down several arithmetical texts, two of them being the Latin and German version of the »Algorism« by Johannes de Sacrobosco, which means that the Latin version is in the same manuscript as its German translation. Cases like this one are perfect to examine transfer processes from Latin to German because a well-known treatise that is stable in terms of its transmission can be found in two languages in one miscellany.[47] We can determine what parts of the Latin text were omitted, changed, or even extended and how technical terms were translated. To illustrate that a little further, I want to give two short examples from the beginning of the treatise:

The Latin version of the Sacrobosco »Algorism« goes into detail about what the term algorism might mean and gives three different explanations. Every interpretation concludes that the word algorism consists of two terms that were later connected and can be translated as »the art of numbers« or »the art of enumeration«.[48] The German translation of this passage works very differently: The three different possible explanations are not mentioned. All we have is a statement that the book is called »die kurzen kunst von der zal« (»the short art of the number«) naming the art after its inventor »Algus«. This is given as a fact, but in the Latin version that is just one of the three possible interpretations of the word *algorismus*. Here we can see that this process of thinking about and explaining terminology, a rather scholarly way of detailing how one comes to certain conclusions, is totally omitted in the German translation. Furthermore, the Latin version in Codex 3502 is heavily annotated. We find Latin glosses everywhere: in the margins of the page on the sides, on the bottom, on top, and interlinear glosses as well. The interlinear glosses comment on single words used in the text, while the glosses in the margins give more information on the content of the treatise. It is a text that has been thought about, worked on, improved, and expanded.

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46 Cf. Habermann, *Deutsche Fachtexte der frühen Neuzeit*; Keil, *Deutsch und Latein*; Fürbeth, Selektion und Transformation.

47 In addition to that, the two treatises need to be examined more closely in terms of paleography. At a first glance it is possible that the same hand wrote the Latin as well as the German version of the Sacrobosco Algorism. Significant letters like »d«, »g« or »p« are executed in the exact same style, and the flow of the writing and the structure of the page are conspicuously similar. It also seems like the initials were not done by a professional illustrator but by the same hand that wrote the running text; those are noticeably similar as well.

48 In the Latin version, which starts on folio 148r, it reads: *Est autem nomen eius algorismus, et dicitur ab »algos«, quod est ars, et »rithmus«, quod est numerus: inde »algorismus« quasi ars numerandi. Vel dicitur ab »an«, quod est in, et »gogos«, ductio, et »rismus«, numerus, quasi inductio in numerum. Tertio modo dicitur ab Algo inventore et »rismus«.*

This treatise, with all its glosses written in several hands, is a subject of scholarly dispute, unlike the German »Algorism«, which is almost not annotated at all. All we can see are a couple of Latin glosses that sometimes re-translate a German term back into Latin, possibly for the sake of accuracy. One example for that is on folio 270r, where the German expressions for the different number categories are given in red ink in Latin in the right margin of the page. This leads to the second observation that concerns the direct translation of Latin mathematical terminology into German. In the Latin text we are introduced to the three categories of numbers. Sacrobosco distinguishes the *digitus*, the single digit, the *articulus*, which is any multiple of ten, and the *numerus compositus*, which defines all other numbers that can be made by joining a *digitus* and an *articulus* together. The German version of the »Algorism« does not use the Latin terminology but translates those terms as »vinger« (»finger«), »geliet« (»limb«), and »gemachte zal« (»done« or »made number«). We can observe the attempt at creating a distinct jargon in the vernacular while building upon the Latin terminology. For the *digitus* the single digit, the »vinger« is a literal translation. The *articulus* translates to »joint« or »knuckle«. The Middle High German word »gelit«[49] mainly refers to a limb but can also mean »joint«. Here the terminology is connected to the process of finger counting: first the whole finger is used to name a single digit, and in a second step, another part of the hand, the knuckle or the joint (specifically the joints of the hand)[50] signifies the multiples of ten. The *numerus compositus* translates to »composite number« and refers to something that is connected from different parts – in our case from a *digitus* and an *articulus*. The German »gemacht« is a difficult translation because the Middle High German »machen«[51] rather means to »create« or »generate«. The Latin meaning is implied and can be explained by the fact that things need to be worked on or »made« up to join them together. Here we can observe this process of concept formation by studying two versions of the same text. The German terminology by itself is not easy to grasp; in connection with the Latin templates the terms make sense and can be situated in the realm of arithmetic. The fact that in the German translation of the »Algorism« the Latin terms for the different number categories are given as glosses on the right margin of the page support this hypothesis that due to the newness of the topic in the vernacular and the lack of an already existing and working terminology, German terminology needs a strong retrospective dependence on the Latin model.

49 On the term »gelit«/»glit« in Middle High German: According to the Middle High German Dictionary by Matthias Lexer, it refers to »glied, gelenk« (Lexer, Middle High German Hand Dictionary, digitized in the dictionary network of the Trier Center for Digital Humanities, version 01/21, accessed on 30 March 2023: woerterbuchnetz.de/?sigle=Lexer#2).

50 Cf. Menninger, *Zahlwort und Ziffer*.

51 The word »machen« takes on several meanings in Middle High German: According to Lexer, Middle High German Dictionary, it refers primarily in a general sense to »hervorbringen, erschaffen, erzeugen (gebären)« (bring forth, create, generate [give birth]), and in a broader sense also to »machen, bewirken, bereiten, anstellen, zu wege bringen« (make, effect, prepare, set up, accomplish); Lexer, Middle High German Hand Dictionary, accessed on 30 March 2023: woerterbuchnetz.de/?sigle=Lexer#3.

But Codex 3502 is also a great example for how this process of conceptualization and the development of a German mathematical jargon succeeds by creating a terminology that becomes an inherent part of the German language. Even more so, we can observe that a certain phrasing becomes part of these arithmetic texts and functions as a structuring tool. In the German Sacrobosco »Algorism« – but also in other German arithmetical texts like the reckoning book Codex 3029[52] – every single species is introduced by first giving the Latin term followed by a German translation and a short explanation of what this species entails. This is the case for every introductory text in German that seeks to explain the basic arithmetic calculations. In Codex 3502 we can read on folio 271v: »Svbstraccio heißt ein vnte ziehung / oder ein abnemung / vnd lert wie man ein zal von der anderen zihen sol oder nemen scholl«[53] First the reader gets a translation of the Latin word *Svbstraccio* and learns that it is a »deduction« or a »removal«. Furthermore, the literal translation is followed by an explanation of what the term means from a practical point of view: In this case a subtraction is defined as the teaching on how to subtract one number from another. This sentence is the beginning of the passage on the species of subtraction and ends one and a half pages later with an explanation of how to check the solution of a subtraction. The next species after that is again introduced using the same pattern and phrasing: »Mediacio heisset ein halbung / vnd lert wie man ein zall in zwey teilen soll [...]« (»Mediacio is called a bisection and teaches how to divide a number in two [...]«).

What we can observe is not only a stable process of translation when it comes to the most important and widespread terms of arithmetic but also a tool to ease the process of composing such educational texts. The fact that translation, explanation and phrasing are intertwined with the textual structure increases the traceability of the passages on the species not only in this codex but in all German and Latin arithmetic texts that follow this structural pattern – and so far, we can state that all of them do. Therefore, the translation process in arithmetic texts is not only of value for the definition of a German mathematical jargon but also serves as a tool to understand the structural framework of pragmatic arithmetic texts of the Late Middle Ages in general.

52 On this topic, see the passage »Theory – Practice« in this paper.

53 This loosely translates to: »A subtraction is a deduction or a removal and teaches how to take away one number from another.«

Theory – Practice

As mentioned at the beginning of this paper, we can see a difference in our sources when it comes to the question of what kind of approach to the topic of arithmetic is taken: We can distinguish between a more theoretical take on mathematics in treatises like the »Algorism« by Johannes de Sacrobosco or a very practical implementation of the rules on arithmetic focusing on examples and model calculation. When we look at our German sources, we can quickly say that in the 15th and 16th centuries the vernacular occupation with arithmetic concentrated on its practical aspect. The Austrian National Library only holds one purely theoretical text on arithmetic in German which is the translation of the »Algorism« by Johannes de Sacrobsco, a text used to teach arithmetic at European universities from the 13th until the 16th century and therefore so ubiquitously known that we still do not know how many copies of the Latin version are extant.[54] We still have seven translations into German[55] and one of them – a text in the miscellany Cod. 3502 – is in Vienna and part of our research corpus as it was written at the end of the 15th century. As we discussed above, other theoretical texts are solely part of larger reckoning books that hold mainly practical arithmetic but give short theoretical introductions when a new basic arithmetic operation is introduced, following the same tradition as the »Algorism«. Therefore, we can identify the theoretical texts in our exemplary corpus as rooted in the Latin algorism tradition as it was used and perpetuated at the universities for nearly 300 centuries: they are translations, abbreviations, and revisions of those well-known algorism treatises. That affinity between the Sacrobosco »Algorism« and the theoretical parts of reckoning books can be verified by looking at two manuscripts of our exemplary corpus: The one stand-alone reckoning book out of the seven sources we investigated (Cod. 3029), that gives information on all the relevant species, deals with integers, and fractions, and also explains the different mathematical »regulae«, uses similar phrasings as the Sacrobosco »Algorism« in Cod. 5277 when a basic arithmetic operation is introduced.

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54 Search results from the Jordanus database lead to the probable assumption that several hundreds of these manuscripts exist.

55 Berlin, National Library, MS germ. fol. 1278 (14th/15th century); Berlin, National Library, MS lat.qu.577 (1408-1428); Munich, Bavarian National Library, Clm 4162 (15th century); Vienna, Austrian National Library, Cod. 3502 (4th quarter of the 15th century); Wrocław, University Library, HS B akc. 1948/207 (15th century); New Haven, Yale Beinecke Library, MS 1024 (14th to mid-15th century); Pommersfelden, Count Schönborns' library, MS 158 (15th century).

Cod. 3029 explains the introduction of the addition as follows: »addirn haist zu sam geben vnd ist / so du ein zal zu der andern gibst« (»to add is to put together and means that you give one number to another«).[56] The »Algorism« in Cod. 5277 phrases the introduction to this species (following the pattern identified in the previous section) as such: »Addicio das ist ein zu gebung / vnd lert wie / man ein zal zw der andern reiten vnd / geben sol« (»Addicio is an addition and teaches how to reckon and give one number to another«).[57] The similarity between those two sentences is evident, differences occur out of the tone of the instruction that correlates with an intended audience and the overall purpose of the texts: The reckoning book is directing its instruction at a reckoner, a reader, who is interacting with the text and all the calculation examples the manuscript gives. It uses a language that sounds like spoken German, full of colloquialisms and direct requests like »sprich« (»say«), »mach« (»do«), or »nym« (»take«) that mimic a real conversation or at least a proper teaching experience. Therefore the »du«, the »you« is used to directly address the recipient who can be identified as the student, learning from a reckoning master who recorded his teachings in this colloquial manner in a book. In the Sacrobosco »Algorism« the reader is, on the contrary, hardly ever addressed directly. The treatise uses a more neutral language and resorts mostly to the indefinite pronoun »man«, or in English: »one«. The sentences are longer and more complex, also mirrored by the use of a more complex vocabulary in the vernacular. Even though both texts follow the same structure and use similar phrasing, we can see that even the more theoretical parts in the reckoning books are subordinated to the overall purpose of teaching practical arithmetic. This is the reason why the language is more direct, more approachable, why the reader is immediately involved, and why the theoretical involvement with arithmetic is hardly worth mentioning.

This short example already shows that the difference between theory and practice is not only relevant in terms of content but also in the way language is used in a text. Another noteworthy first observation concerns bilingual texts that deal with mathematics. Codex 5277 transmits an algebra treatise in Latin and German written in the first half of the 16th century. The text is written by one hand and functions as an instruction about algebra. It is set up like all functional mathematical texts are structured, with an introduction to the different basic calculation operations followed by examples that convey the practical side of the teaching. That is not particularly special, but Codex 5277 gives its rather extensive theory in Latin and most of its examples in German, therefore dividing the text not only into a theoretical and a practical part but also distinguishing between an academic and a vernacular mode of teaching that directly correlates with the former: theory and the scholarly language Latin are as connected as the practical side of mathematics and the vernacular German.

56 Vienna, Austrian National Library, Cod. 3029, fol. 2r.
57 Vienna, Austrian National Library, Cod. 5277, fol. 277.

Due to the great number of German reckoning examples, we can determine that the target audience for most of the texts in our corpus must have been students or merchants on the one hand, but also people working in trade or adjacent fields who needed to use practical arithmetic daily on the other hand:[58] the reckoning examples in our exemplary corpus can serve as a pool of templates when they mathematically introduce the most important arithmetic operations but also, in terms of content, cover a lot of situations in which mathematical skills were required for a merchant or working commoner. We can read about trading different goods (spices, livestock, metals…) at different European markets with one or more trading partners. Among many other topics, we learn about how to start a business with one or more partners, how to calculate interest when going into debt or when lending money, how to make a profit from buying and selling, and how to barter without getting scammed. This very practical side of arithmetic is also strongly connected to the region where the reckoning book or the example calculations were used: examples work with regional currencies and places of trade as well as trade routes, and the traded goods correlate with the actual geographical reality and show that reckoning books are more strongly connected to and give more information on cultural and social history than we would assume at first glance.[59]

58 In her project »Geometria Deutsch. Druckwerke der praktischen Geometrie bis gegen Ende des 16. Jahrhunderts«, Christina Lechtermann determined that when it comes to printed works of geometry, the practical use of those texts was only one of many purposes: the aspect of collecting books and building a collection, of the aesthetics of printed texts, and also the philological aspects are as much and sometimes even more so part of the use and function of practical geometry as the didactic purpose of the printed book (see the project website for further information: gepris.dfg.de/gepris/projekt/381793985/ergebnisse).

When we look at handwritten arithmetic texts in miscellanies, we can determine that the practical use of those texts was sometimes not the main reason for collecting them: the books can be considered collections of knowledge, e.g. in a university setting; they can also serve the purpose of commenting on other texts when, for example, German and Latin treatises are transmitted in one miscellany and both texts are referencing each other. One difference we can observe when it comes to handwritten mathematical texts in contrast to the printed books, is that the collection and the aesthetics aspect of stand-alone reckoning books is not as important as it is going to be for the printed versions later on. There, mistakes in examples are often reproduced, leading to the question of whether the books were actually used to learn practical arithmetic or whether they fulfilled a different purpose, just as the printed geometry treatises studied in the »Geometria Deutsch«-project suggest. Handwritten stand-alone books are often not »pretty«; they show significant signs of use; they come in a small format; and they are heavily glossed, corrected and added to. We can therefore assume that at a time before the printing press solved the problem of accessibility to teaching materials, those sources of practical arithmetic did actually fulfill the purpose of teaching the skill of mathematics; the strong focus on problems of trade and commerce does suggest, that the target audience for those specific reckoning books was (future) merchants, craftsmen and people working in adjacent fields.

59 Wiesinger, Regula virginum.

Mnemotechnical Aspects

The sources of our exemplary corpus show one additional very interesting aspect we want to discuss further: In Codex 5184, a Latin miscellany composed of texts that are rooted in the quadrivium, we can only find three small snippets of German. One of those short texts, added after a Latin text on geometry, is only a couple of verses long, written in a mixture of Latin and German, and refers to the shape of the Hindu-Arabic numerals by connecting them to objects of utility. We know of at least 22 codices[60] that contain that small text, which can be identified via its incipit »vnvm dat vinger«, »the finger gives you the one«.[61] This already shows how the process of referencing works: a known object, which is given in German, that resembles the shape of a Hindu-Arabic numeral is connected to said numeral, which is sometimes introduced in Latin, sometimes in German. The finger is connected to the symbol for the number one; stairs or a crutch[62] are the reference object for the two; the pigtail is the shape connected to the Hindu-Arabic numeral three, and so forth. Some of the reference objects are of a rather curious nature or can even be considered a little bold: the symbol for the number zero, the very well-known circle, is in several manuscripts introduced as »bruochring«, which translates to the small loops in a medieval pair of underpants, through which some kind of string was threaded and tied up in order to keep the garment from slipping down the body. Even mentioning underpants in the Late Middle Ages can be considered a breach of taboo. At that time underpants were a solely male item of clothing and in addition to that, they were also part of the private realm. Talking about or even showing them was scandalous and usually connected to deliberate provocation or even used in a sexualized way.[63] Using such funny, weird, or even outrageous images to recall the shapes of the Hindu-Arabic numerals is therefore interesting and bold – but nothing new. It follows a well-known tradition that both Mary Carruthers and Frances Yates explain in their standard works on the ancient and medieval art of memory.[64] They can show that already in ancient Greece and Rome, the memory was believed to be reinforced by triggering emotional connections.

60 The 22 manuscripts are: Annaberg-Buchholz Erzgebirgsmuseum M 001; Basel University Library FVIII 16; Basel University Library FVII 12; Berlin National Library Ms. Lat. Qu. 2; Dessau City Library GB 866.8 Grad (Georg HS); Dresden Saxonia State Library C 80; Erfurt Bibliotheca Amploniana Ampl. Oct. 80; Gotha Research Library Chart B. 445; Graz University Library 275; Kremsmünster Monastery Library fragment; Leipzig University Library 1470; Michelstadt Evangelische Kirchenbibliothek D 692; Munich Bavarian National Library Clm 24539; Munich University Library 4 Cod. Ms 649; Salzburg St. Peter B IX14; Salzburg St. Peter B III 32; Straßburg Bibliothèque Nationale et Universitaire C 102 (burnt); Vatikan/Heidelberg Vat. Pal. Lat 1452; Weimar Bibliothek der nat. Forsch. u Gedenkstätte O 110e; Vienna Austrian National Library 5184; Wolfenbüttel Herzog August Library Cod. Guelf. 1189 Helmst.; Wolfenbüttel Herzog August Library 16.1 Astronom 4°.

61 A monograph written by Michaela Wiesinger and Christina Jackel on this mnemonic is currently on its way and will be published in 2024/25.

62 The reference objects for the shape of the numerals are not stable; several objects (the finger for the number one, the pigtail for the number three can be found in all known versions of the mnemonic, but the other objects of reference vary throughout the manuscripts. There could be several reasons for this: The shapes of the Hindu-Arabic numerals were not stable throughout the Late Middle Ages and the early modern period. We can observe a change in how the numbers were recorded in different areas and at different times until the invention of the printing press brought easier reproducibility and therefore a certain stability. The changing shapes could have been mirrored by changing reference objects in the mnemonic; more so because our 22 sources have been put down in writing over the span of 250 years. Another possible explanation for the changing reference objects could be a misreading or a new interpretation of a template. We can see that in case of the zero, when the »ring« is changed into the »brouchring« to add a new and very specific meaning.

63 Nutz and Stadler, Gebrauchsgegenstand und Symbol; Jaritz, Die Bruoch.

64 Cf. Carruthers, *Book of Memory*; Yates, *Art of Memory*.

A mental marker does not need to be pretty or even elegant, it needs to work, and the more absurd, the more violent, grotesque or even obscene the marker, the more likely our brains do not forget. Carruthers explicitly says that those markers can even overlap to form one big image that holds all the necessary information to remember a certain set of things. She says that »The disgusting and the silly, the noble and the violent, the grotesque and the beautiful, the scatological and the sexual [...]«[65] can all be utilized to generate an image or images that help us to remember.[66] That seems to be the case when it comes to the »vnvm dat vinger«. The funny, absurd, and vulgar images are certainly a way to avoid forgetting the mnemonic itself, but also a way to avoid forgetting the items that need to be remembered: in our case the shape of the Hindu-Arabic numerals and therefore the numerals themselves.

Another application of mnemonic tactics can be observed in Codex 3528. The miscellany holds 10 pages of reckoning examples that mainly address the rule of three. On several of those 10 pages we can find small illustrations in the margins right next to the reckoning examples. Every image is linked to one specific example, usually depicting the object that is the pivot of said example. When livestock is traded, a little sheep adorns that part of the page. When a house is bought, we can see the image of a small house; the buying and selling of goods in barrels is accompanied by an illustration of a barrel. Even though the images are pretty and make the text look more interesting, there is more to them than just sprucing up the page. Collections of reckoning examples in manuscripts or even sparsely distributed examples throughout a miscellany are given for a reason: They are sample problems that touch on different mathematical problems but mainly cover different economic settings and several trades with different goods. The reckoning examples in Codex 3528 are all calculated carefully: they do not just give the problem and the solution; the solution path is also part of every example. This could mean that this collection of reckoning examples served as a kind of template for other problems with similar methodological and content-related issues. Having a small pool of examples as reference for additional mathematical problems is advantageous in terms of the practical application of calculation rules. Using a template saves time and is especially useful in an economic setting because actual transactions and deals call for quick problem solving and good calculation skills. The little images next to the reckoning examples in Codex 3528 could easily serve that purpose: to help remember a sample calculation that deals with skills needed to solve problems of similar content. Remembering the traded goods mentioned in those examples is easier than remembering the reckoning method itself because an image of a wheel or of a piece of fruit is more tangible than the rule of three on its own. The illustrations serve as mnemonic devices that conceptually connect the content of a reckoning example with the mathematical skills needed to solve the given problem. In terms of practical implementation that could mean, for example, to tackle a mathematical problem like the »sheep example«, invoking a reckoning method via a mnemonic marker, in this case a small and funny looking illustration.

65 Cf. Carruthers, *Book of Memory*, 171 (see footnote 61)

66 Cf. Carruthers, *Book of Memory*, 168-184 (see footnote 61)

Conclusion
Editing, describing, and analyzing almost 140 texts is not a small feat. Our chosen approach of starting with a small exemplary corpus to test our initial hypotheses, find feasible workflows, and show first results has proven valuable. It showed us in more detail what we can expect from our sources in terms of problems, questions, and feasibility. It also led to the identification of the main areas of research we need to deal with in the process of this research project: First, there is the beginning of an evolution of a mathematical jargon in the vernacular that we will study in combination with the Latin sources handed down with the German texts. Second, we have found that our focus on the development and diffusion of practical knowledge in order to distinguish it from the Latin theoretical tradition will be a major field of investigation that contributes to the current research focus in the field of history of science focusing on practical knowledge.[67] And third, we cannot underestimate the cultural and social aspects that led to the formation of this very distinct category of practical literature, created its own discourse, and mutually refers to and incorporates other discourses.

But looking at our exemplary corpus not only helped to carve out our overall research questions in more detail but also contributed to testing and sharpening our methodological approach: we were able to agree upon a functioning method to describe our corpus in the necessary detail without having to produce in-depth catalog entries. We were also able to test out and get to know our transcription software Transkribus and establish a very useful HTR model specifically for mathematical texts to help with the transcription process.

Having reached these first integral goals, we will now tackle the edition and description of the rest of the corpus, starting in chronological order. It is necessary to generate a stable and sound edition as a coherent basis for our further contextual and content-related research. With an increasing number of edited primary sources, we are hoping for an even more detailed and better suited process of description as well as a consistently refined HTR model due to the ever-growing number of transcribed pages we can add to the existing model as further training data.

Overall, we can say that especially at the beginning of a research project dealing with a large and pretty much unknown corpus, the examination of a small exemplary corpus was immensely helpful in focusing our attention on the process of methodology development and analytical approach. Our first findings are not only exciting and motivational but also serve as a basis for our further research on the topic, creating a baseline for broader questions that might even be more interdisciplinary in nature.

67 A new book on that topic was just published: Smith, *From Lived Experience to the Written Word*.

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Abbreviations
ÖNB = Austrian National Library
ZIM = Zentrum für Informationsmodellierung, Austrian Center of Digital Humanities, University of Graz

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Basel University Library FVII 12
Berlin National Library Ms. Lat. Qu. 2
Berlin, National Library, MS germ. fol. 1278 (14th/15th century)
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København, Kgl. Bibl., Ny kgl. Saml. 275a,40 (late 13th century)
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